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Quantitative implementation of Preisach-Mayergoyz space to find static and dynamic elastic moduli in rock

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Abstract. In this paper we describe the analysis of quasi-static stress-strain data using a Preisach-Mayergoyz (PM) [after *Preisach*, 1935; *Mayergoyz*, 1985] space picture for the elastic behavior of rock. In contrast to the traditional analytic approach to stress strain (an energy density as a function of the strain invariants), the PM space picture reproduces hysteresis and discrete memory seen in the data. In addition, the PM space picture establishes a relationship between experimental data and a number density ρ of microscopic mechanical units within the rock. The density ρ allows us to make quantitative predictions of dynamic elastic properties. Determining ρ from quasi-static stress-strain data requires us to solve a highly underdetermined inverse problem. We explore the following three methods of solving the inverse problem: simulated annealing, normal modes, and exponential decay. All three methods are tested on a Berea sandstone data set and found to give an excellent description of stress versus strain. Choosing one method, the normal mode method, we analyze quasi-static stress-strain curves on two additional sandstones, namely, another sample of Berea and a sample of Castlegate sandstone. From the density ρ for each sample we predict the dynamic modulus as a function of pressure and the nonlinear elastic constants. For each of these cases the agreement between the predictions based on ρ and experiment is quite good. We establish that PM space provides a quantitative description of the elastic response of a rock and that PM space may be found by a variety of inversion methods.

Introduction

This paper is the second in a series on the elastic properties of rocks. In the first paper [*McCall and Guyer*, 1994] we introduced a description of rock elasticity that accounts for observed history and memory features. The central construct in the description was the Preisach-Mayergoyz (PM) space [after *Preisach*, 1935; *Mayergoyz*, 1985] in which the macroscopic material response is tracked using the density ρ of microscopic mechanical units (the density in PM space). *McCall and Guyer* [1994] showed that from ρ one could learn the elastic properties of the rock (the quasi-static modulus, the dynamic modulus, the strength of the cubic and quartic nonlinearities, etc.). In addition, we suggested that quasi-static stress-strain measurements make it possible to learn ρ. Thus the recipe for the elastic properties of a rock is (1) collect quasi-static stress-strain data, (2) invert for ρ (PM space), and (3) predict elastic properties. The purpose of this paper is to demonstrate the determination of ρ from quasi-static stress-strain data. We call this the inverse problem. We use data on three sandstone samples called B1, B2, and C. We use the ρ that we find for these samples to determine their linear and nonlinear elastic properties.

We begin with a section reviewing the traditional theory of elastic wave propagation in nonlinear materials. Then we review the principal results of application of the PM space picture. Particular attention is given to the results leading to formulation of the inverse problem. Three methods for solving the inverse problem are described and applied to quasi-static stress-strain data on the B1 sample. We choose one of these methods, the method of normal modes, and apply it to all three data sets. We find ρ for each data set and show how ρ is used to predict the behavior of quasi-static stress-strain curves, predict the dynamic modulus, and predict the linear and nonlinear elastic coefficients.

Review of the Traditional Theory

The traditional theory of elastic wave propagation in a nonlinear material is based on expressing the energy density as a function of the scalar invariants of

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